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TRANSIENT SOLIDIFICATION OF A FLOWING LIQUID AT A HEAT CONDUCTING WALL

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Abstract — The solidification of a flowing liquid at the heat conducting wall of a channel or a tube is investigated. The initial temperature of the wall is below the solidification temperature of the liquid, whereas the temperature of the liquid at the channel entrance is higher than the solidification temperature. While penetrating into the channel, the liquid is cooled due to heat transfer from the liquid to the solidified layer. On the other hand, the temperature of the channel rises due to heat transfer from the solidified layer to the channel wall. Both effects are taken into account in the analysis. If the wall temperature increases to values above the solidification temperature of the liquid, the solidified layer eventually disappears locally.

NOMENCLATURE

A, area of channel cross section;
a, thermal diffusivity;
b, channel width;
c, specific heat capacity;
D, dimensionless parameter, cf. equation (25);
K, dimensionless parameter, cf. equation (14);
l, longitudinal coordinate in the channel (arc length);
N, dimensionless parameter, cf. equation (26);
Nu, Nusselt number;
q, heat flux density;
T, temperature;
t, t', global and local time, respectively;
U, perimeter of channel cross section (heat conducting part only);
V, channel volume;
$\dot{V}$, volume flow rate of liquid;
v, flow velocity of liquid;
x, coordinate normal to wall;
Z, dimensionless parameter, cf. equation (28).

Greek symbols

Δ, dimensionless thickness of solidified layer, cf. equation (11);
δ, thickness of solidified layer;
ε, penetration depth;
ζ, dimensionless length coordinate; cf. (11);
θ, dimensionless temperature difference, cf. equation (11);
λ, heat conductivity;
ρ, density;
τ, τ', dimensionless global and local time, respectively; cf. (11) and (19).

Subscripts

L, liquid material;
0, initial or entrance conditions;
S, solid material; also: state of solidification;
W, wall;
∇, free surface of rising liquid;
$*$, reference quantities.

1. INTRODUCTION

IN CASTING technology the following problem arises. A liquid penetrates into the channel of a mould. The mould is initially at a temperature below the solidification temperature of the liquid. At the cold wall the liquid solidifies. The solidified layer not only threatens to block the channel flow but may also be the reason for undesirable forces in the mould due to thermal stresses.

Problems of that kind have already been the subject of many investigations. A general introduction into the field of solidification is given in [1], and a collection of more recent papers is given in [2] and [3].

More specifically, Beaubouef and Chapman [4] analysed the freezing of a solid phase from a fluid flowing past a cold surface. The wall temperature was assumed to be constant, and the surface convective heat flux was prescribed. K. Stephan and his co-workers [5–7] extended the problem considerably by considering the flow in a tube and taking the ambient heat transfer into account.

Blockade due to solidification in the flow of a saturated liquid through channels and into cavities was investigated by J. Madejski [8]. Three flow conditions were considered in [8], namely that of constant velocity, and those of constant pressure drop in laminar and turbulent flow, respectively. The penetration of a liquid at its solidification temperature into a tube that is maintained at a constant temperature below the solidification temperature was also studied by Epstein, Yim and Cheung [9]. They obtained an approximate solution by postulating a reasonable functional form for the instantaneous shape of the solidified layer. In a later paper [10], Epstein and Hauser checked the assumption on the layer shape by a numerical solution of the full integrodifferential equation governing the liquid motion. In this context it might be of interest to note that an exact analytical solution for the shape of a two-dimensional solidified layer on a constant-temperature plate of finite width was obtained by Siegel [11] with the help of conformal mapping methods.

Liquid solidification in the laminar entrance flow of a circular tube with uniform wall temperature was considered by Hwang and Sheu [12]. Uniform wall temperature was also assumed in the investigations on steady-state freezing in turbulent flow inside tubes [13, 14] as well as in a recent study on transient freezing [15].

In this paper the problem of solidification of a liquid at the wall of a channel or a tube is, in accordance with the requirements of some applications, stated somewhat differently. When the liquid enters the channel, the temperature of the liquid is higher than the solidification temperature. While penetrating into the channel, the liquid is cooled due to heat transfer from the liquid to the solidified layer. On the other hand, the temperature of the channel, being initially at a value below the solidification point of the liquid, rises due to heat transfer from the solidified layer to the channel wall. Both effects strongly influence the shape and the thickness of the solidified layer. If it happens that the wall temperature increases during the process to values above the solidification temperature of the liquid, the solidified layer eventually even disappears locally.

2. TEMPERATURE CHANGE IN THE RISING LIQUID

Consider a liquid (heat conductivity λ_L) rising in a channel (width b) bounded by a heat conducting wall (heat conductivity λ_W) on one side and an adiabatic wall (or an axis of symmetry) on the other side (Fig. 1 and Fig. 2, upper part). Let the initial temperature of

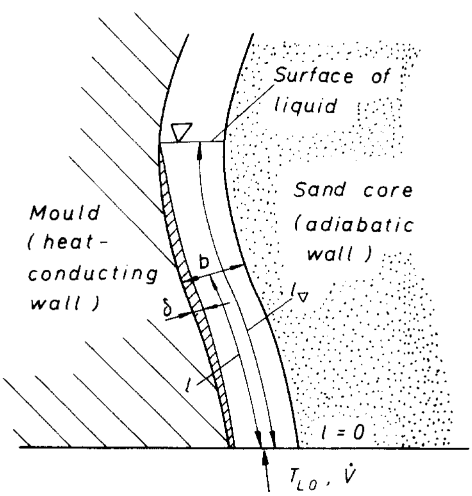

FIG. 1. Rising liquid and solidified layer in a channel with the temperature of one wall being below the solidification temperature.

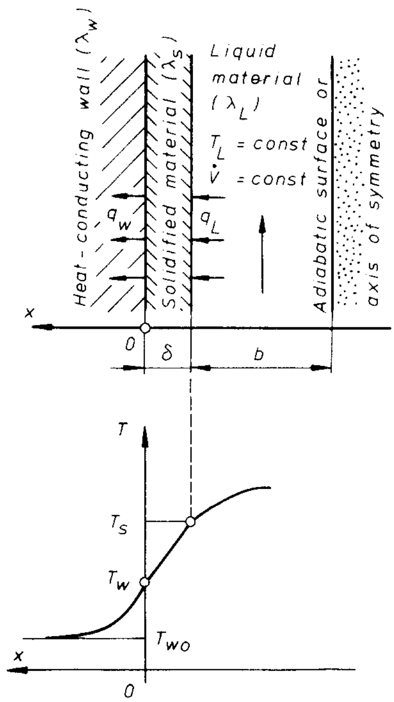

FIG. 2. Solidification with constant heat flux from the liquid.

the heat-conducting wall, T_{WO}, i.e. the temperature of the wall before the surface of the rising liquid arrived at the point under consideration, be smaller than the solidification temperature T_S of the liquid, but the bulk temperature of the flowing liquid, T_L, larger than T_S.

A layer of solidified material (heat conductivity λ_S) is formed at the conducting wall such that the temperature at the boundary between the solid and the liquid is equal to the constant value T_S (Fig. 2, lower part). The solidified layer is assumed to be very thin in comparison with the channel width b.

If the flow at the mould entrance (cf. Fig. 1) is at a steady state, i.e. the volume flow rate $\dot V$ and the entrance temperature of the liquid T_{L0} are independent of time, and if, furthermore, the heat-conducting wall is totally covered by the thin solidified layer with constant surface temperature T_S, then the bulk temperature of the liquid, T_L, as well as the heat flux from the liquid to the solidified layer are independent of time, too, and depend only on the local position (given by the path length l, cf. Fig. 1).

The heat flux density from the liquid to the solidified layer can be expressed in terms of the Nusselt number Nu according to the relation

$$q_L = Nu\lambda_L(T_L - T_S)/2b. \quad (1)$$

Thus the energy balance for the liquid yields

$$-\dot V \rho_L c_L \frac{dT_L}{dl} = \lambda_L(T_L - T_S)\frac{Nu\,U}{2b} \quad (2)$$

where ρ_L and c_L are, respectively, the density and the specific heat conductivity of the liquid, and U is the perimeter of the heat-conducting wall in a horizontal cross-section at the position l. In general, U as well as b and Nu will depend on l.

The solution of the differential equation (2) subject to the boundary condition $T_L = T_{L0}$ at $l = 0$ is

$$\frac{T_L - T_S}{T_{L0} - T_S} = \exp\left[-\frac{a_L}{2\bar{V}}\int_0^l \frac{NuU}{b}\,d\bar{l}\right]. \qquad (3)$$

Here, the thermal diffusivity of the liquid, $a_L = \lambda_L/\rho_L c_L$, has been introduced.

3. THE LOCAL SOLIDIFICATION PROCESS

The thickness of the solidified layer, δ, as well as the wall surface temperature T_W change with time. In order to simplify the problem we consider the case that not only the solidified layer thickness but also the penetration depth with respect to the temperature disturbances in the wall is very small in comparison with the channel width b. Thus the theory of one-dimensional heat conduction can be applied [16]. We furthermore assume that the internal energy change of the solidified material is negligible. This assumption is justified if $c_S(T_S - T_W) \ll r_S$, where c_S is the specific heat capacity of the solid material, and r_S is the specific melting enthalpy. It follows that the temperature distribution in the solidified layer is a linear one (cf. Fig. 2).

An integral method is applied to the unsteady heat conduction process in the wall by integrating the heat conduction equation with respect to the space coordinate x and approximating the temperature distribution by the parabolic profile

$$T - T_{W0} = (1 - x/\varepsilon)^2 (T_W - T_{W0}) \quad \text{for } 0 \leqq x \leqq \varepsilon,$$

and

$$T - T_{W0} = 0 \quad \text{for } x > \varepsilon.$$

Obviously, ε characterises the penetration depth with respect to the temperature disturbances.

By this procedure we obtain the following set of equations which represent, in this order, the conservation of energy in the wall, at the interface between wall and solidified layer, and at the interface between solidified layer and liquid:

$$\rho_W c_W \frac{d}{dt'}[\varepsilon(T_W - T_{W0})] = 6\lambda_W (T_W - T_{W0})/\varepsilon \qquad (4)$$

$$\lambda_S (T_S - T_W)/\delta = 2\lambda_W (T_W - T_{W0})/\varepsilon \qquad (5)$$

$$\lambda_S (T_S - T_W)/\delta = q_L + \rho_S r_S \frac{d\delta}{dt'}. \qquad (6)$$

t' is the time passed since the free surface of the liquid arrived at the point under consideration, and c_W is the specific heat capacity of the wall material, ρ_W and ρ_S are the densities of the wall and the solidified material, respectively, and r_S is the melting enthalpy per unit mass of the solidified material. The other symbols have already been introduced above.

According to the definition of time t', the initial conditions at any level of the channel are

$$\delta = 0, \quad \varepsilon = 0 \quad \text{at } t' = 0. \qquad (7)$$

Combining the equations (1) and (4)–(6), integrating once with respect to time, and eliminating ε, we obtain the following system of equations:

$$\theta = 1 + K(1 + \tau'/\Delta) \times \{1 - [1 + 2/K(1 + \tau'/\Delta)]^{1/2}\} \qquad (8)$$

$$\frac{d(\Delta^2)}{d\tau'} = 2(\theta - \Delta) \qquad (9)$$

$$\Delta = 0 \quad \text{at } \tau' = 0 \qquad (10)$$

where the dimensionless variables

$$\theta = \frac{T_S - T_W}{T_S - T_{W0}}, \quad \Delta = \frac{\delta}{\delta^*}, \quad \tau' = \frac{t'}{t^*} \qquad (11)$$

have been introduced with the reference quantities

$$\delta^* = \frac{2b}{Nu}\frac{\lambda_S}{\lambda_L}\frac{T_S - T_{W0}}{T_L - T_S} \qquad (12)$$

$$t^* = \left(\frac{2b}{Nu}\right)^2 \frac{\lambda_S \rho_S r_S (T_S - T_{W0})}{\lambda_L^2 (T_L - T_S)^2} \qquad (13)$$

and the dimensionless parameter

$$K = \frac{3\lambda_S \rho_S r_S}{4\lambda_W \rho_W c_W (T_S - T_{W0})}. \qquad (14)$$

An asymptotic expansion of equations (8) and (9) for small times yields

$$\theta = \theta_0 = 1 + K[1 - (1 + 2/K)^{1/2}]$$
$$\Delta = (2\theta_0 \tau')^{1/2} \quad \text{as } \tau' \to 0+. \qquad (15)$$

This result is of a more general importance in the limiting case of fluid temperature being equal to the solidification temperature. Since $t^* \to \infty$ as $T_L \to T_S$, equation (15) already provides the solution in this limiting case for all finite times t'. Therefore, if $T_L = T_S$, the wall temperature T_W remains constant for all positive, finite times, whereas the thickness of the solidified layer increases proportional to the square-root of time.

The system of equations (8)–(10) has been solved numerically by a Runge–Kutta method. Results are given in Figs. 3 and 4. Of particular interest is the time-dependence of the thickness of the solidified layer at a given position (given value of l), cf. Fig. 3. After a rather fast increase from zero to a maximum value, the layer thickness decreases, and eventually the solidified layer vanishes. This is due to the fact that the wall temperature increases with time as a consequence of the heat flux towards the wall and, after a certain time, its value becomes equal to the solidification temperature T_S; cf. Fig. 4.

4. ANALYTICAL SOLUTION FOR $K \to \infty$

Let us recall now that our investigation is based on the assumption that the solidified layer is very thin in

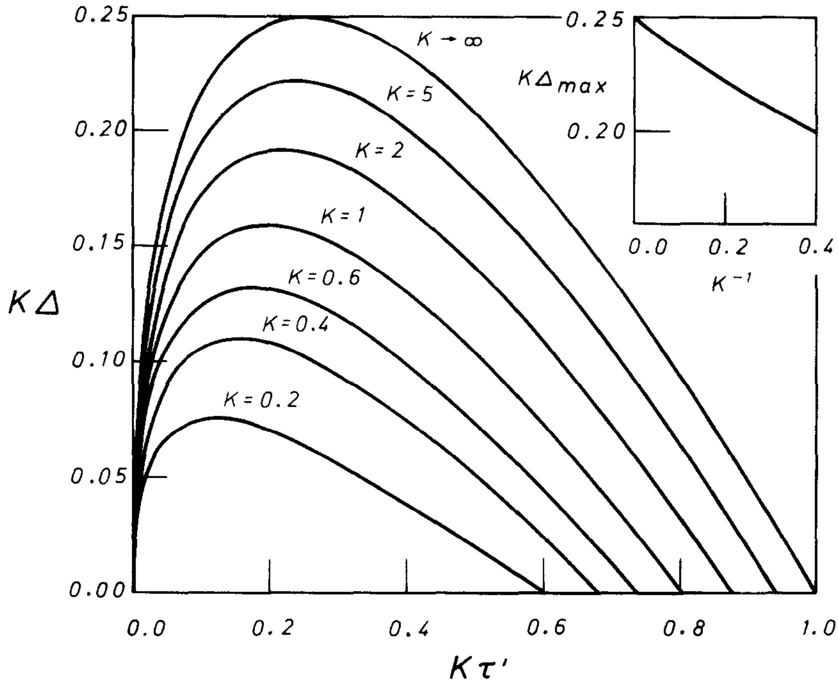

FIG. 3. Dimensionless thickness of the solidified layer as a function of the dimensionless local time.

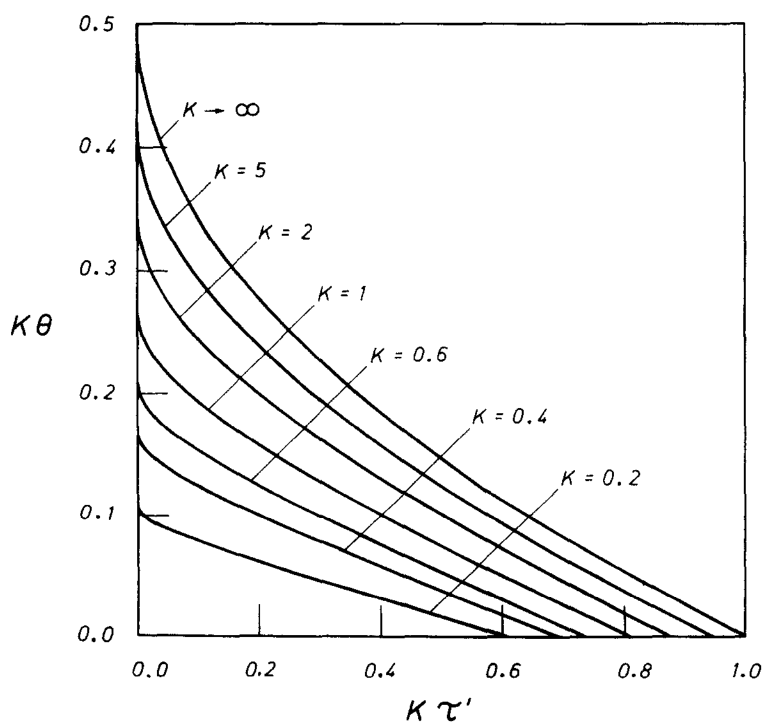

FIG. 4. Dimensionless wall temperature as a function of the dimensionless local time.

comparison with the channel width b. The reference value of the layer thickness δ^* is given by equation (12). Since the heat conductivities of the solid and the liquid material are certainly of the same order of magnitude, and since we can furthermore assume that in most cases of practical interest the Nusselt number will be of order 1, the ratio δ^*/b will be of the order of $(T_S - T_{W0})/(T_L - T_S)$. This ratio is usually also of order 1 or even larger.

If all the assumptions mentioned above are satisfied it follows that δ^* is of the order of b, or even larger. Furthermore the solutions given in Fig. 3 show that the dimensionless layer thickness $\Delta = \delta/\delta^* = O(1)$ if $K = O(1)$. But $\Delta \to 0$ as $K \to \infty$, and it is therefore this limiting case which is of interest in many applications.

Expanding the system of equations (8)–(10) in terms of K^{-1} we can solve the differential equation for the first-order terms and obtain the following solution for $K \to \infty$ and $0 \leqq K\tau' \leqq 1$:

$$K\Delta = (K\tau')^{1/2} - K\tau'$$
$$K\theta = \tfrac{1}{2}[1 - (K\tau')^{1/2}]. \tag{16}$$

This solution is also shown in Figs. 3 and 4. Note that $\Delta = O(K^{-1})$ and $\theta = O(K^{-1})$.

5. RELATIONS BETWEEN LOCAL TIME t' AND GLOBAL TIME t

Wall temperature and thickness of the solidified layer depend on position (coordinate l) and time. At a given position, the time-dependence of wall temperature and layer thickness is given by the solution (16) in terms of τ', which is a dimensionless 'local' time defined such that $\tau' = 0$ (and $t' = 0$) when the free surface of the rising liquid arrives at the point with coordinate l. Hence the origin of the local time variable τ' (and t') depends on the position of the point under consideration.

Since it is desirable to know the spatial distribution of wall temperature and layer thickness at one and the same moment of time, we shall introduce a 'global' time t such that $t = 0$ when the free surface is at $l = 0$ (i.e. when the liquid enters the mould; cf. Fig. 1).

If $\dot{V}$ is the constant volume flow rate, and $V(l)$ is the volume of the channel of length l (Fig. 1), then local time and global time are related to each other by the equation

$$t - t' = V(l)/\dot{V}. \tag{17}$$

Furthermore, since $t' = 0$ at the liquid free surface which is located at $l = l_\nabla$ (Fig. 1), the global time t can be expressed in terms of the volume occupied by the liquid:

$$t = V(l_\nabla)/\dot{V}. \tag{18}$$

The dimensionless local time τ' has been defined by referring t' to the reference time t^* which, according to equation (13), depends on the local quantities b, Nu, and T_L. Therefore t^* is not suited to be a reference time for the global time, and we rather introduce a dimensionless global time τ as follows:

$$\tau = t/t_0^* \tag{19}$$

with

$$t_0^* = t^*\big|_{l=0} = \left(\frac{2b_0}{Nu_0}\right)^2 \frac{\lambda_S \rho_S r_S (T_S - T_{W0})}{\lambda_L^2 (T_{L0} - T_S)^2}, \tag{20}$$

where the subscript 0 at the quantities b, Nu, and T_L refers to the mould entrance conditions ($l = 0$).

In terms of dimensionless times, equation (17) becomes

$$\tau - \left[\frac{bNu_0(T_{L0} - T_S)}{b_0 Nu(T_L - T_S)}\right]^2 \tau' = \frac{V(l)}{\dot{V} t_0^*}. \tag{21}$$

This is the desired relation between the dimensionless local and global times.

6. RESULTS FOR A VERTICAL, CYLINDRICAL CHANNEL

Consider a vertical, cylindrical channel of constant width b (Fig. 5). Apart from obvious smoothness requirements, the cross-section may be of an arbitrary (simply or doubly connected) shape. Since perimeter U, cross-section area A, flow velocity $v = \dot{V}/A$, and Nusselt number Nu are independent of the length coordinate (height) l, the following expression for the local temperature distribution in the liquid is obtained from equation (3):

$$\frac{T_L - T_S}{T_{L0} - T_S} = \exp\left(-\frac{a_L Nu\, U}{2vA}\frac{l}{b}\right). \tag{22}$$

In order to obtain the thickness of the solidified layer as a function of local position and time, we have to express the dimensionless local time τ' in terms of given quantities. With $V/\dot{V} = l/v$ and $\tau = t/t_0^* = l_\nabla/vt_0^*$, we obtain from equation (21)

$$\tau' = \frac{l_\nabla - l}{vt_0^*}\left(\frac{T_L - T_S}{T_{L0} - T_S}\right)^2. \tag{23}$$

Introducing equations (22) and (23) into equation (16), rewriting $\Delta = \delta/\delta^*$ with δ^* given by equation (12), and using the expression (14) for K, we finally obtain for

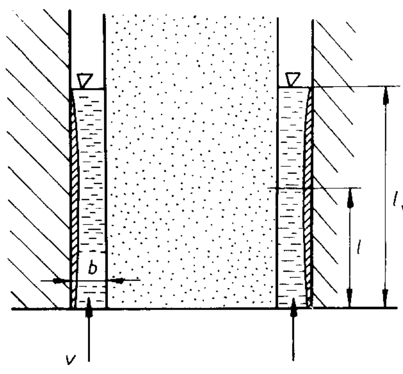

FIG. 5. Vertical, cylindrical channel.

the ratio of solidified layer thickness δ and channel width b the following equation:

$$D\frac{\delta}{b} = (\zeta_V - \zeta)^{1/2} - (\zeta_V - \zeta)\,e^{-N\zeta}. \qquad (24)$$

Here D and N are dimensionless parameters given by

$$D = \frac{3Nu\lambda_L\rho_S r_S(T_{LO} - T_S)}{8\lambda_W\rho_W c_W(T_S - T_{WO})^2}, \qquad (25)$$

$$N = \frac{8\lambda_W\rho_W c_W(T_S - T_{WO})^2}{3Nu\lambda_L\rho_L c_L(T_{LO} - T_S)^2}\frac{Ub}{A}. \qquad (26)$$

Furthermore, ζ is a dimensionless length coordinate defined by

$$\zeta = Zl/b \qquad (27)$$

with

$$Z = \frac{3\lambda_L^2}{\lambda_W\rho_W c_W}\left(\frac{Nu}{4}\frac{T_{LO} - T_S}{T_S - T_{WO}}\right)^2\frac{1}{vb}, \qquad (28)$$

and ζ_V, the ζ-value at the free surface of the liquid, represents a dimensionless global time according to the relation

$$\zeta_V = Zl_V/b = Zvt/b. \qquad (29)$$

Note that for a thin channel, Ub/A can be replaced by 1. In this case no geometrical quantities appear in the parameter N.

Using equation (24), the distribution of the thickness of the solidified layer has been calculated for $N = 0.5$. The results are given in Fig. 6.

Remarkably, the solidified layer profile is qualitatively different at different times. For rather small dimensionless times the layer thickness increases monotonically from zero at the free surface to the value at the entrance cross section of the channel; cf. the curve for $\zeta_V = 0.2$ in Fig. 6. For intermediate dimensionless times the layer thickness has a maximum value somewhere in the channel but is still non-zero at the entrance cross-section; cf. the curves for $\zeta_V = 0.4$–0.8. At time $\zeta_V = 1$ the solidified layer vanishes at the entrance cross-section, and for even larger times part of the heat-conducting wall is wetted with the liquid without a solidified layer in between. This is due to the fact that the wall temperature increases above the solidification temperature. Since the calculation of the temperature of the liquid is based on the assumption that the wall is totally covered by a solidified layer, cf. Section 2, the solution is, strictly speaking, not valid any more if $\zeta_V > 1$. However, it can be used as an approximate solution provided that the uncovered part of the wall is very small in comparison with the total length of the liquid-filled channel.

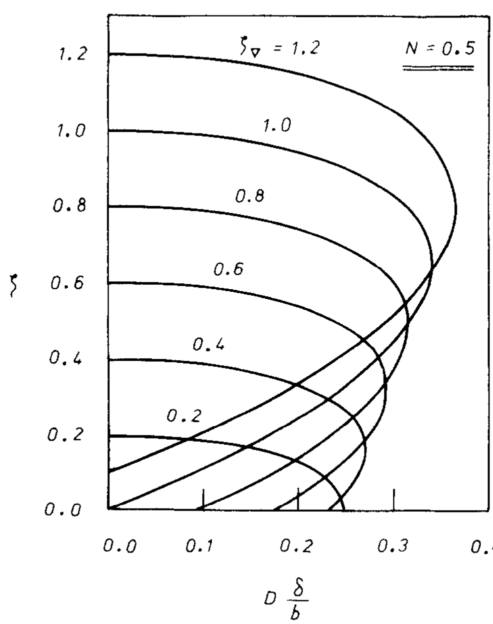

FIG. 6. Dimensionless thickness of the solidified layer as a function of the dimensionless length-coordinate, with the dimensionless global time as parameter.

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SOLIDIFICATION TRANSITOIRE D'UN LIQUIDE S'ECOULANT CONTRE UNE PAROI THERMIQUEMENT CONDUCTRICE

Résumé—On étudie la solidification d'un liquide en écoulement contre la paroi d'un canal ou d'un tube. La température initiale de la paroi est inférieure à la température de solidification du liquide, tandis que la température du liquide à l'entrée est supérieure à cette dernière. En pénétrant dans le canal, le liquide est refroidi du fait du transfert de chaleur entre le liquide et la couche solidifiée. D'autre part, la température du canal s'élève du fait du transfert de chaleur de la couche solide à la paroi du canal. Les deux effets sont pris en compte dans l'analyse. Si la température de la paroi s'élève à des valeurs supérieures a la température de solidification du liquide, la couche solidifiée disparait éventuellement localement.

INSTATIONÄRES ERSTARREN EINER STRÖMENDEN FLÜSSIGKEIT AN EINER WÄRMELEITENDEN WAND

Zusammenfassung — Das Erstarren einer strömenden Flüssigkeit an einer wärmeleitenden Wand eines Kanals oder eines Rohrs wird untersucht. Die Anfangstemperatur der Wand liegt unterhalb der Erstarrungstemperatur der Flüssigkeit, während die Flüssigkeitstemperatur am Kanaleinlaß höher als die Erstarrungstemperatur ist. Als Folge des Wärmeübergangs von der Flüssigkeit auf die erstarrte Schicht wird die Flüssigkeit während des Eindringens in den Kanal abgekühlt. Andererseits steigt die Temperatur des Kanals zufolge des Wärmeübergangs von der erstarrten Schicht auf die Kanalwand. Beide Effekte werden in der Rechnung berücksichtigt. Wenn die Wandtemperatur auf Werte oberhalb der Erstarrungstemperatur der Flüssigkeit ansteigt, verschwindet die erstarrte Schicht schließlich örtlich.

НЕСТАЦИОНАРНЫЙ ПРОЦЕСС ЗАТВЕРДЕВАНИЯ ЖИДКОСТИ ПРИ ТЕЧЕНИИ ВДОЛЬ ТЕПЛОПРОВОДНОЙ СТЕНКИ

Аннотация — Исследуется затвердевание жидкости при течении вдоль теплопроводной стенки канала или трубы. Начальная температура стенки ниже, а температура жидкости на входе в канал выше температуры ее затвердевания. В канале жидкость охлаждается за счет переноса тепла к затвердевающему слою. В то же время стенка канала нагревается за счет потока тепла от затвердевшего слоя. При анализе учитываются оба эффекта. При достижении стенкой температуры, превышающей температуру затвердевания жидкости, затвердевший слой со временем локально исчезает.